

# Bayesian Decision Models for Environmental Risks

## *Modelli di decisione Bayesiani per l'analisi di rischi ambientali*

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**Riassunto:** L'analisi statistica di rischi ambientali è caratterizzata da notevole complessità anche in relazione a strutture di dipendenza dei dati. Inoltre è spesso richiesto di andare oltre l'inferenza, per valutare le conseguenze di decisioni e raccomandare strategie risolutive. La metodologia Bayesiana ha dato prova di saper fornire strumenti molto flessibili sia per l'inferenza che per l'analisi decisionale nello studio di problemi complessi, e la tipologia dei modelli gerarchici si dimostra uno strumento congruo sia per analisi decisionali a livello locale che per la valutazione di politiche a livello generale. Il problema dell'avvelenamento da arsenico dei pozzi per il rifornimento idrico che colpisce vaste aree del Bangladesh è illustrato come caso esemplare che richieda la costruzione di un modello flessibile e un'analisi delle politiche di riduzione del rischio.

**Keywords:** environmental statistics, spatial statistics, decision analysis, fully hierarchical Bayesian analysis.

## 1. Introduction

Modern Bayesian analysis may provide statisticians with powerful tools for facing complex real problems. There is nothing (or almost nothing) new in saying that Bayesian approach often overcomes the frequentist one in offering a wider range of solutions for data modeling. Instead, it has been less explored its potential of fully combining the modeling phase (or more generally statistical inference) with the decision-making stage.

Most statisticians (Bayesians and non Bayesians), in fact, end their analyses once a model (which is thought good enough for their purposes) has been built and inference made. Nonetheless, statistical analysis is often implicitly or more explicitly asked to go beyond inference: it is essentially asked to predict what will be eventually observed or shall never be. One form of prediction consists in extending inference beyond observed data, to unobserved units, missing observations, unobserved outcomes for different factor levels, etc. The other form of prediction consists in extending inference for estimating the consequences of decision options and for eventually giving valuable recommendations. This latter form, which is known as decision-making process, is the one we are here interested in.

Although some statisticians defend the inference-only approach, especially by pointing out the extreme difficulty in choosing meaningful and appropriate measures for the consequences (or utility functions that are the very last summary of such measures), decision-making is sometimes the focussed purpose of an analysis. In this case, some authors sustain that modern Bayesian inference is ideally suited for blending with formal decision-theoretic tools, both from a conceptual standpoint and for its powerful computation tools, the Markov chain Monte Carlo methods. Even, they sustain, both inference and decision-

making can take advantage from each other by such a blending.

For all the foregoing arguments, environmental statistics is one of the many applied settings, such as public health, medicine as well as social sciences, which could greatly benefit from using modern Bayesian methods. In fact, statistical analysis of environmental hazard problems is often quite challenging. First, it involves making inference in complex dependence structures such as geologic/geophysical/geochemical systems, spatial/spatio-temporal, socio-economic territorial settings. Further, it is generally asked to go beyond inference for carrying out risk assessment and decision making.

Section 2 illustrates some relevant concepts of a fully hierarchical Bayesian decision analysis. The motivating application, consisting in the problem of widespread arsenic poisoning of groundwater wells in Bangladesh, is described in Section 3, at the end of which some currently effective solutions for arsenic exposure reduction are summarized. Section 4 concludes with posing the directions of the current and future research steps toward a fully hierarchical Bayesian decision analysis of this environmental problem.

## **2. A fully Bayesian analysis**

This section illustrates some advantages in using Bayesian methods both for inference and decision analysis. An unifying approach, that is a fully Bayesian analysis, is then advocated especially for complex real problems like the environmental studies typically are.

### **2.1 Bayesian modeling of complex systems**

Statistical analysis of environmental risk problems often involves modeling spatial (or spatio-temporal) dependence structures, which are but one feature of these however complex systems. As Banerjee *et al.* (2004) declare in the preface to their book, traditional material to deal with spatial data (the basic reference of which is the textbook of Cressie (1993)) is

wanting in terms of flexibility to deal with realistic assumptions. Traditional Gaussian kriging is obviously the most important method of point-to-point spatial interpolation, but extending the paradigm beyond this was awkward. For areal [...] data, the problem seemed even more acute: CAR models should most naturally appear as priors for the parameters in a model, not as a model for the observations themselves.

Fully hierarchical Bayesian spatial models show then to be flexible enough to account for point and/or area-level dependence and for misaligned spatial data as well, within however highly-structured systems.

A part from being more flexible in modeling, the Bayesian approach may be favoured by a further aspect: it enables incorporating any information and/or expert belief, available prior to the current study, by opportunely specifying the prior distributions. Most highly structured models can be barely identified or even not identified (Gelfand and Sahu (1999)). Then, the use of informative priors, whenever any substantial information is available, can be quite convenient.

On this respect, any time informative priors are chosen, it is a good practice to conduct a sensitivity analysis, i. e. repeat the procedure with different priors and see what effects

(if any) they have on the posterior outcomes. Such a practice is a simple remedy for one of the weaknesses most often imputed to Bayesian methods: their subjectivity due to dependence on prior distributions (Berger (1990)). Nonetheless, this is a useful practice in decision analysis context as we argue later (Stangl (1995)).

## 2.2 Bayesian hierarchical decision analysis

The approach pursued in this paper is a fully hierarchical Bayesian decision analysis the basis of which have been laid by the earliest work of Stangl (1995), more extensively by Lin *et al.* (1999), coming up more recently to Coull *et al.* (2003). It essentially consists in using a hierarchical Bayesian model as the “prior probability model” to probabilistic decision analysis. That is, once a formal decision analysis has been set up (generally by a decision-uncertainty tree which encompasses all the possible decision options and associated outcome variables), predictive distributions resulting from a hierarchical Bayesian model are used to define the decision outcome probability distributions. Thereby, model predictive distributions enter decision analysis as prior distributions for the typical risk calculations, that is averaging over uncertainties as well as optimization over decisions. Moreover, Bayesian inference enters within decision analysis whenever uncertainty tree is of multi-stage kind: predictive distributions are then updated conditionally to the given information of earlier decisions. The term “prior”, perhaps ambiguously used above, is so clarified: predictive distributions are posterior to survey data but are prior to any information collected at interim decision stages.

But why should a blend of Bayesian inference with Bayesian decision analysis be beneficial? And, in particular, why should hierarchical methods?

The first question is promptly answered. In the standard Bayesian approach to decision analysis, decision options are evaluated in terms of their expected outcomes, averaging over a probability distribution which summarizes the state of knowledge prior to decision-making. The probability distribution is typically extrapolated either from expert opinion elicitation or from literature review or even from summary estimates of a previously fitted model. Then, a first benefit of such a blend is the one commonly imputed to fully Bayesian analyses: the advantage of building a single unifying model enabling all sources of variability and uncertainty to correctly propagate throughout its levels, this time incorporating also decision making stages. Furtherly, (intimately related to this argument is that) a blend makes much more transparent the process by which a decision-maker eventually comes to certain risk evaluations and decision recommendations. In an unifying process, the prior inputs to decision analysis are well defined by setting up a probability model with clear assumptions on parameter priors, model likelihood and possible utility functions.

On this regard, it is worth noting that the benefit is both in one way and in the opposite way. Final decisions can be made much stronger (and possibly more justified) by clearly defining all the model inputs to decision process. Nonetheless, final decisions themselves can be used as model checks (Gelman *et al.* (1996)) (are the recommended actions under the posited model consistent with what would be expected from the data and the specific purposes at hand for decision-making?) and so inform the modeling itself.

Sensitivity analysis, that is, decision sensitivity to model choice and priors in this context, is strongly needed here. The question is no more (or not only) that of testing inference robustness, though that of correctly mapping decision inputs (parameter priors, likelihood models, utility functions) to the recommended outputs (Stangl (1995), Gelman *et al.*

(2003)).

We lastly hint that fully Bayesian decision analysis for multi-stage trees makes feasible optimal sequential monitoring for numbers of interim looks that are untenable (by the process known as backward induction) in traditional approaches (Berry and Ho (1988), Carlin *et al.* (1998)).

Finally, the specific advantages of hierarchical models are mentioned. By hierarchical model, we practically mean a statistical model in which each unit of interest is individually modeled, conditionally to the underlying population model. Hence, it allows individually varying inference and locally calibrated decision recommendations. An individual/local decision process can then be more finely tuned by a hierarchical model than by a traditional nonhierarchical one. On this respect, environmental problems, often featuring a noticeable spatial variability, may be more opportunely tackled with a hierarchical decision strategy.

In addition, also decision policies adopted for the entire population can be more correctly assessed by opportunely aggregating the individual effects of actions taken at local level (Lin *et al.* (1999)).

Introductory books to theoretical issues in decision theory and the connection to Bayesian inference are Savage (1954), Luce and Raiffa (1957), DeGroot (1970), Berger (1985). In environmental literature, applied works specifically related to this connection, are Wolpert *et al.* (1993), Taylor *et al.* (1993), Brand and Small (1995), Dankins *et al.* (1996), Lin *et al.* (1999), Pool and Raftery (2000), Raftery (2003), Coull *et al.* (2003). Parmigiani (2002)) is a textbook on medical decision-making, whose clinical trials is a long-standing application (Berry and Ho (1988), Carlin *et al.* (1998), Parmigiani (2004)). Other examples are in social applications (Gelman *et al.* (2003))) and in generally named cost-benefit analyses. These references do not pretend to exhaust all the most important and/or recent works on the topic. However, still, risk assessment and decision-making literature seem in need of more formal Bayesian modern thinking.

### 3. An environmental application

The problem of arsenic in drinking water from tube wells in Bangladesh (Gelman *et al.* (2004)) serves as a paradigmatic example to illustrate typical issues featuring environmental hazards studies. The usefulness of a fully hierarchical Bayesian decision analysis will be made apparent, by facing each relevant issue, in the present and in the concluding discussion.

Many of the wells used for drinking water in Bangladesh and other South Asian countries are contaminated with natural arsenic, affecting an estimated 100 million people. Arsenic is a cumulative poison, and exposure increases the risk of cancer and other diseases.

Two datasets are available for this study: a small dataset from an intensive local study, providing information on arsenic level, depth, number of users, year of installation and geographical positioning for each well on a small area of a region of Bangladesh (~5000 wells over ~20km<sup>2</sup> by 2000 augmented to ~6000 wells over ~25km<sup>2</sup> by 2001, of Araihasar *upazila*; Figure 1); a large dataset from a World Bank-sponsored survey providing information on the same variables as above but the last one, for a major portion of wells over the entire region (~30000 wells over ~440km<sup>2</sup>; Figure 2). As for the latter dataset, only the geographical positioning of a major part of centroids of large areas partitioning the region is known (for 144 over the 176 *mouzas* consisting each of a group of villages).

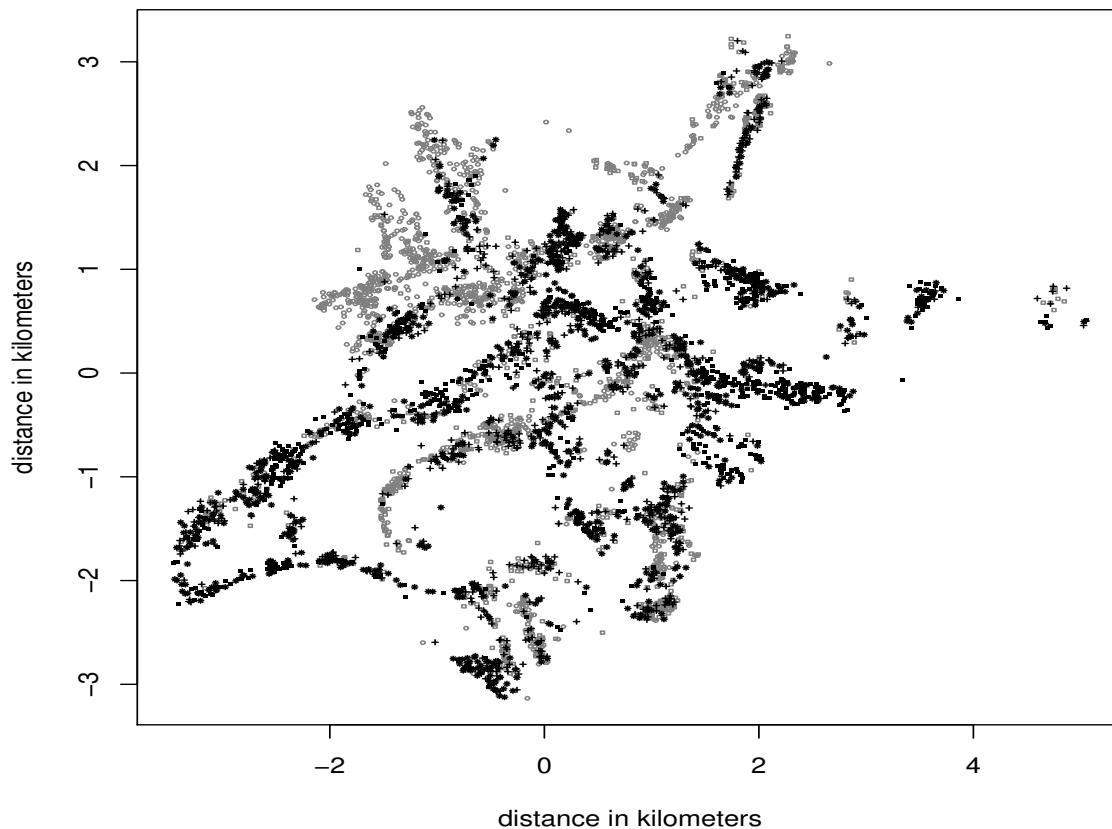


Figure 1: *Tube wells in a section of Araihaazar Upazila, Bangladesh. (The (0, 0) point on this graph is at latitude 23.8° north and longitude 90.6° east.) Each dot represents a well, and these are all the wells in this area. Symbols indicate arsenic levels: gray circle (less than 10  $\mu\text{g/L}$ ), gray square (10–50), cross (50–100), asterisk (100–200), and black filled square (over 200). By comparison, the maximum recommended levels designated by Bangladesh and the World Health Organization are 50 and 10, respectively.*

Figure 1 is a map of arsenic levels in all the wells of the completely covered small area in Araihaazar: circle and square lighter dots are the safest wells, cross and asterisk dots exceed the Bangladesh standard of 50 micrograms per liter, and filled square dots indicate the highest levels of arsenic. Safe and dangerous wells are then intermingled. Things are really more complicated than this because the depth of the well is an important predictor, with different depths being safe “zones”, or layers, in different areas (Figure 3 shows this relation but for the entire region). Same evidence derives from inspection of the larger dataset, though spatial variability can be here observed only at an aggregated (village within *mouza* and *mouza*) level. In summary, arsenic concentrations are extremely variable within lateral and vertical spatial scales.

The purpose of this study is finding strategies for arsenic exposure mitigation. A short-term decision, for people who are currently drinking from high-arsenic wells, essentially consists in switching to nearby low-arsenic wells. We note that this option can be undertaken under certainty only in the smaller area, since there are unobserved wells in the larger area whose arsenic levels are unknown. This short-term option has been exhaustively studied in Gelman *et al.* (2004) and the aggregated effects on the average arsenic

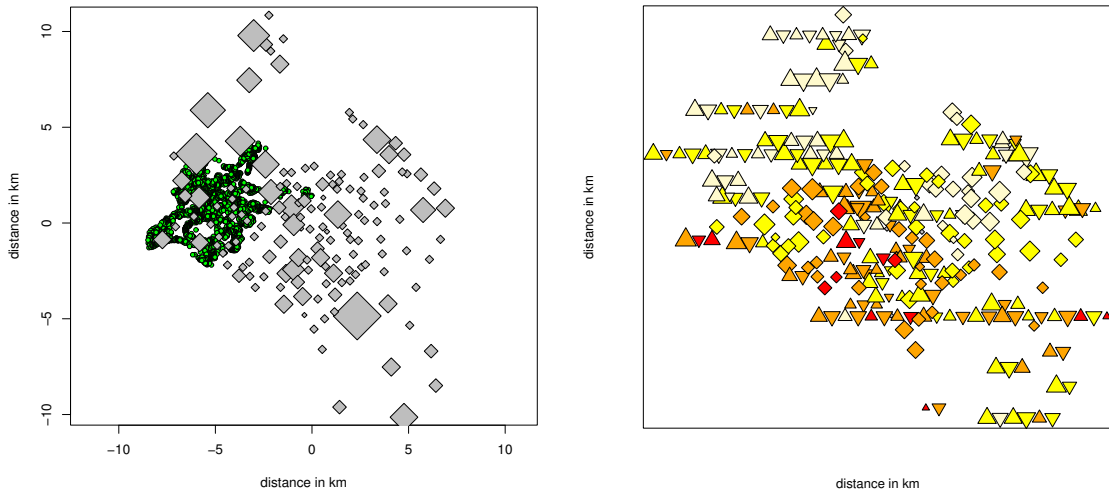


Figure 2: Araihaazar upazila, Bangladesh: (left) location of the all ~5000 individual wells (in a western small area) and location of 149 mouzas (symbol size is proportional to the number of surveyed villages grouped into each mouza); (right) Proportion of unsafe wells in the 302 villages:  $[0, 0.1]$ ,  $(0.1, 0.5]$ ,  $(0.5, 0.9]$  and  $(0.9, 1]$  are indicated by increasingly darker grays. Village location is the same as the mouza one: the number of surveyed wells per village ranges from 1 to 565 with a median around 70 (diamonds are used when we have only one village per mouza, otherwise the villages are depicted as a number of horizontally aligned up- and down-triangles; symbol size is proportional to a function of the number of surveyed wells per village).

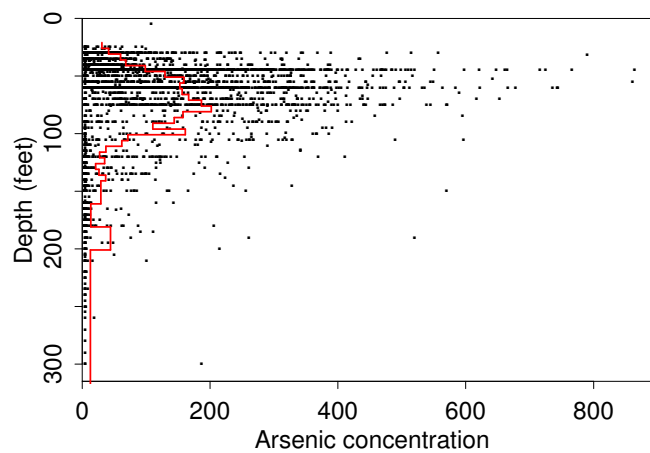


Figure 3: Arsenic concentrations and depths of the ~5000 wells mapped in Figure 1. The superimposed line shows average arsenic concentration as a function of depth.

exposure of all the area residents have been evaluated according to several assumptions on the nearest safe well distance, safe well overburdening rates, and positioning of possibly new wells. However, this option has been currently discarded since it is a solution of short-term and is unfeasible in areas with high rates of unsafe wells (not less for the unpredictable effects on arsenic levels of safe overburdened wells). Thus, (after discarding a quite expensive solution consisting of an arsenic removal system) the installation of new wells tapping safe aquifers, a medium-term option, still provides the most effective means of reducing arsenic exposure.

Purpose of inference is then estimating, for any location within a high arsenic exposure area, an arsenic-safe depth zone, whether exists, at which drilling a new well. Yet, once we had a model for making such an inference, would the statistical job be really finished? At this point, enough insight has been attained into the problem so that we can recognize the many statistical issues that we generally discussed in former sections.

**A complex system** The two available datasets feature any kind of spatial dependence. The first one consists of point-referenced whereas the second one of areal (or block) spatial data. Misaligned spatial data arise as well, when we combine the two datasets for the overlap villages of the smaller area. Moreover, the estimation of arsenic-safe depth layers is on its own a non-standard inferential issue.

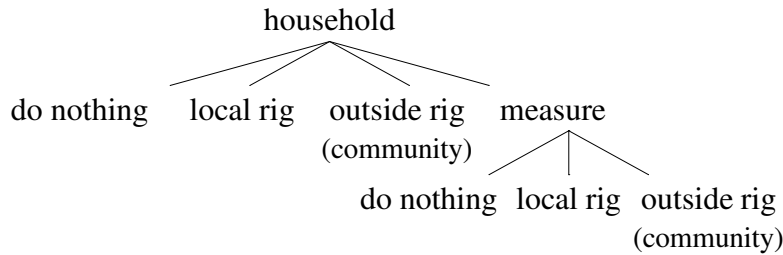
It essentially consists in dividing the depth range (from 0 to 300 *ft* on average; see *y*-axis scale of Figure 3) of any interest-location  $s$  into safe and unsafe (low- and high-arsenic concentration, respectively) layers. What we know is, for each surveyed well, whether is safe and how much deep it is. However such an information is spatially available in two different ways: we can count on individual information for  $\sim 6000$  sites  $s'$  across the smaller area, only on a block-information for  $\sim 150$  *mouzas* across the entire *upazila*. This latter information has been currently augmenting: block data will be available for almost all 300 villages (*mouza* sub-areas) by 2005 autumn.

The pattern found by empirical data inspection (Figure 3)—that safe depth “zones” are likely to be either the shallowest depths (from surface up to 30/40 *ft*) or the deepest ones (below  $\sim 100$  *ft* on average)—has been convincingly associated with local stratigraphy as well as geological and chemical characteristics by recent studies (van Geen *et al.* (2003)). Unfortunately, groundwater of shallow wells is a source of further concern since, besides arsenic, other contaminants can have an elevated concentration. Hence, identifying, for any given interest-location, a safe-depth threshold below which groundwater arsenic levels are typically low, seems to be the only left solution. However, geological and chemical spatial patterns, besides being still imperfectly understood, are also quite variable. Inspected data show that such a safe-depth might be varying from about 70 *ft* to well below 300 *ft* (Figure 4 of Gelman *et al.* (2004), not reported here) across the region. A model is needed that finely integrate any kind of expert knowledge with all of the patterns emerging from empirical data.

**A decision-making problem** We premise that the vast majority of existing tube wells in Bangladesh were paid by individual households and therefore are privately owned (van Geen *et al.* (2003)). However, the maximum depth accessible to local drilling technology is 300 *ft*, beyond which the provision of an outside rig is needed. In this case, decision whether to afford much higher costs is likely to be taken by a community of people or households.

Suppose that a household is able to know groundwater arsenic concentration  $y$  at any

depth  $d$  at the location  $s$  where it lives,  $y = f(d, s)$ . Then, possible decision options are: (1) do nothing, (2) drill a new well by a local rig at a depth  $< 300\text{ft}$ , (3) drill a new well at a depth  $\geq 300\text{ft}$  by affording the cost of an outside rig, possibly passing through a community agreement. In decision-making under certainty, households will decide for the least expensive solution so to have an arsenic level at  $50\mu/\text{lg}$  at maximum. Suppose, instead, that a household is given only a probability distribution of  $y$  conditional to  $d$  as well as  $s$ ,  $p(y|d, s)$ . Then, decision-making under uncertainty involves a further decision option, that is (4) take a measurement by paying an *ad-hoc* equipment. In conclusion, this decision analysis can be represented by the following tree-diagram,



which is a typical two-stage tree. At the first stage, the measurement action is chosen when the so called “value of information” is greater than the expected utility evaluated for the other three options.

Local decision process has now been clarified, and a hierarchical spatial Bayesian model for inference on  $p(y|d, s)$  seems the most appropriate solution for the reasons earlier explained. Decisions must be made locally, nonetheless the assesement of effects at regional level is important as well for providing guidelines to public health and social policies. It can be properly evaluated by, first, fixing idealized recommendations at local level and, then, making all the local effects confluence to the aggregated consequence of interest.

### 3.1 The current effective solution

Gelman *et al.* (2004) provides a nonparametric solution for identifying a safe-depth for the smaller area dataset. This method is well far from the sophisticated modeling idea that is the object of the current research. However, its results seem to have convinced the earth scientists of the team and, even, its implementing algorithm has nowadays been running through an automated system to provide such estimates for the entire region.

In the current solution, safe-depths are separately estimated for each “small area”. In Gelman *et al.* (2004) such small areas consist in clusters of wells as produced by a well-known clustering method, the  $k$ -means algorithm, by using solely the spatial coordinates of each point-well. As for the larger dataset, such small areas are made corresponding to villages, the sub-areas partitioning the region.

The inferential method, implemented by an algorithm called *search algorihm*, has been constructed, on one side, “mimicing” the patterns se see in in the data (Figure 4 of the reference article; Figure 3), on the other hand, having in mind a problem of statistical decision theory. The result can be effectively described by a tree diagram where, roughly, each branch represents the search of the first two deepest undafe wells for the small area considered. Safe-depth threshold is then defined as the depth, below the second deepest unsafe well if any, which maximizes the probability that a a well drilled deeper than



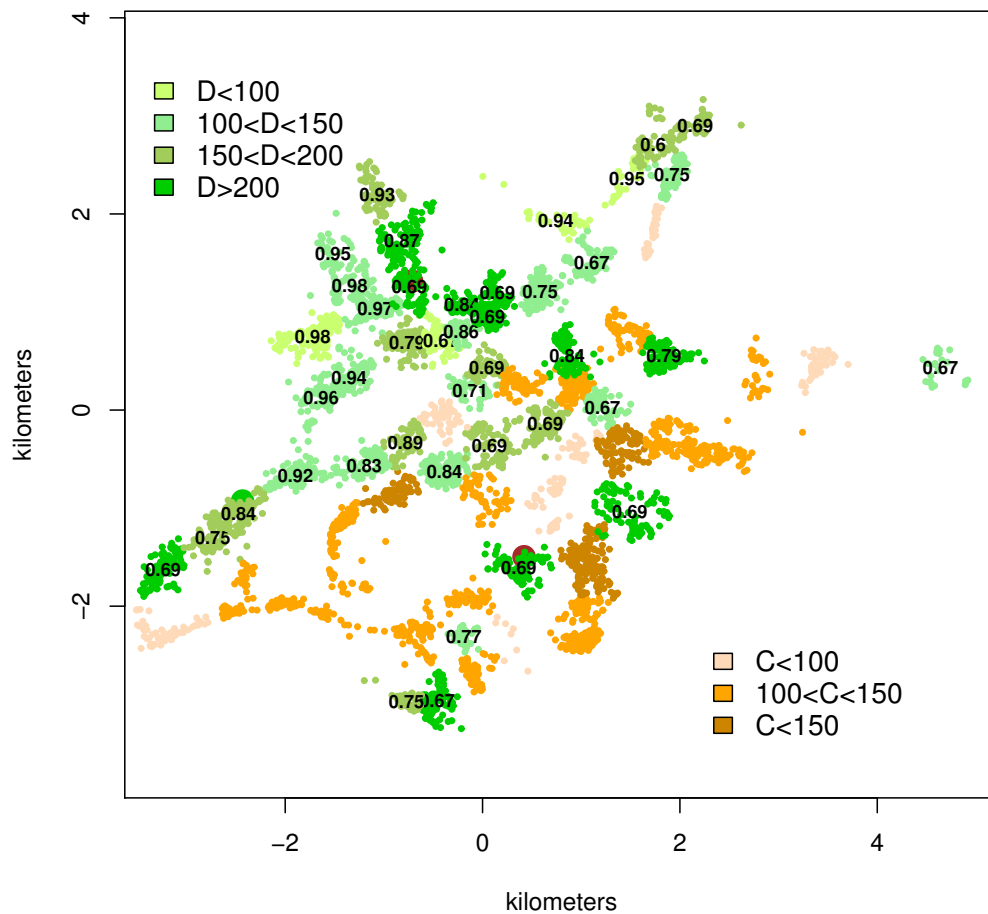


Figure 4: *Estimated safe depth thresholds, and estimated probabilities that a new well will be safe, if it is deeper than the estimated safe depth, within each of the spatial clusters of the smaller area. Estimated depth thresholds  $D$  are indicated by different shadows. The clusters with bounds on depth thresholds can be distinguished by non having such probability indication and the censored bounds  $C$  are indicated by different shadows.*

it has actually low-arsenic level. This probability is estimated according to a semiparametric Bayesian method that updates fairly informative priors conditional to the number of safe wells observed below the candidate threshold. These uncertainty estimates are then calibrated by a cross-validation procedure (Figure 14 of the reference article). A second algorithm, called *matching algorithm*, which implements an alternative Bayesian semiparametric method this time closer in spirit to statistical hypothesis testing, confirms the results obtained by the first method.

If we consider the smaller area, the results can be displayed by a map such as the one of Figure 4. The safe-depth threshold and probability estimates are then conveyed through a special network—described in the concluding section—to the region residents who will be helped in taking the proper decision for tackling the arsenic crisis.

## 4. Future research

Fifty community wells and a number of private wells have been installed since 2001 by Columbia University and its local partners of Bangladesh in the smaller area of Arai-hazar (Figure 1), by using the results obtained from the recent studies that we have so far discussed. The newly installed wells have become extremely popular and have led to a drastic reduction in exposure of the population to arsenic. The current proposal is to build on this experience by expanding the scale of the intervention to the entire region of Arai-hazar (Figure 2) and, afterwards (from 2006, conditional to the re-funding of the project), throughout the country. The challenge will be to maximize the impact of the intervention by taking advantage of the larger World-Bank sponsored survey dataset (that will eventually cover 5 millions of wells from 2006) which will be augmented with data collected during the installation of new wells. Several local employees with experience in catalyzing community participation at the village level are currently trained to interpret the available results in a spatial context. Each employee is provided with a cellular phone and a hand-held GPS receiver. The technology gives access from any of the ~300 villages of Arai-hazar to the automated calculation of the likely depth of a safe aquifer at that location based on the information stored in a central data base and an indication of the uncertainty of the estimate. Moreover, this technology provides a means of instantly updating the safe-depth estimate by uploading new data from any location and stores the coordinates and characteristics of newly installed wells.

The task assigned to the statisticians has several steps. First, it involves building a hierarchical model for estimating safe-depths and safety probabilities at village level. This is currently carried out by a two level hierarchy with first level modeling safety probability conditional to depth and safe threshold, second level modeling safe threshold, and hyperprior level incorporating the expert knowledges. A spatial hierarchical structure for misaligned data is assigned to threshold second level model wherever point-referenced data are available (so far limited to the overlap smaller area villages). Information on contiguity structure of villages (whose centroid location is gradually being loaded by the automated system) will be eventually considered in the model. Second, a decision-making set up has to be carefully specified so as to evaluate aggregated effects at regional level of local decisions taken by the villagers helped in by local employees. Once these steps are accomplished, a fully Bayesian decision analysis is ready to be carried out, together with an accurate sensitivity analysis on priors, likelihoods and any utility functions.

## References

- Banerjee S., Carlin B.P. and Gelfand A.E. (2004) *Hierarchical Modeling and Analysis for Spatial Data*, Chapman & Hall.
- Berger J.O. (1985) *Statistical Decision Theory and Bayesian Analysis*, New York: Springer-Verlag.
- Berger J.O. (1990) Robust bayesian analysis: Sensitivity to the prior, *Journal of Statistical Planning and Inference*, 25, 303–328.
- Berry D.A. and Ho C. (1988) One-sided sequential stopping boundaries for clinical trials: A decision-theoretic approach, *Biometrics*, 44, 219–227.

- Brand K.P. and Small M.J. (1995) Updating uncertainty in an integrated risk assessment: conceptual framework and methods, *Risk Analysis*, 15, 719–731.
- Carlin B.P., Kadane J.B. and Gelfand A.E. (1998) Approaches for optimal sequential decision analysis in clinical trials, *Biometrics*, 54, 964–975.
- Coull B.A., Mezzetti M. and Ryan L.M. (2003) A bayesian hierarchical model for risk assessment of methylmercury, *Journal of Agricultural, Biological & Environmental Statistics*, 8, 253–270.
- Cressie N.A.C. (1993) *Statistics for Spatial Data*, New York: Wiley.
- Dankins M.E., Toll J.E., J. S.M. and Brand K.P. (1996) Risk-based environmental remediation: Bayesian monte carlo analysis and expected value of sample information), *Risk Analysis*, 16, 67–79.
- DeGroot M.H. (1970) *Optimal Statistical Decisions*, New York: McGraw-Hill.
- Gelfand A.E. and Sahu S.K. (1999) Identifiability, improper priors, and gibbs sampling for generalized linear models, *J. Amer. Statist. Assoc.*, 94, 247–253.
- Gelman A., Meng X. and Stern H. (1996) Posterior predictive assessment of model fitness via realized discrepancies (with discussion), *Statistica Sinica*, 6, 733–807.
- Gelman A., Stevens M. and Chan V. (2003) Regression modeling and meta-analysis for decision-making: a cost-benefit analysis of incentives in telephone surveys, *Journal of Business and Economic Statistics*, 21, 213–225.
- Gelman A., Trevisani M., Lu H. and van Geen L. (2004) Direct data manipulation for local decision analysis, as applied to the problem of arsenic in drinking water from tube wells in bangladesh, *Risk Analysis*, 24, 1597–1612.
- Lin C., Gelman A., Price P.N. and Krantz D.H. (1999) Analysis of local decisions using hierarchical modeling, applied to home radon measurement and remediation (with discussion), *Statistical Science*, 14, 305–337.
- Luce R.D. and Raiffa H. (1957) *Games and Decisions*, New York: Wiley.
- Parmigiani G. (2002) *Modeling in Medical Decision Making*, New York: Wiley.
- Parmigiani G. (2004) Uncertainty and the value of diagnostic information, *Statistics in Medicine*, 23, 843–855.
- Pool D. and Raftery A.E. (2000) Inference for deterministic simulation models: the bayesian melding approach, *J. Amer. Statist. Assoc.*, 95, 1244–1255.
- Raftery A.E.a. (2003) Bayesian uncertainty assessment in multicompartament deterministic simulation models for environmental risk assessment, *Environmetrics*, 14, 355–371.
- Savage L.J. (1954) *The Foundations of Statistics*, New York: Dover.
- Stangl D.K. (1995) Prediction and decision making using bayesian hierarchical models, *Statistics in Medicine*, 14, 2173–2190.
- Taylor A.C., Evans J.S. and McKone C.E. (1993) The value of animal test information in environmental control decision, *Risk Analysis*, 13, 403–412.
- van Geen A., Zheng Y., Versteeg R., Stute M., Horneman A., Dhar R., Steckler M., Gelman A., Small C., Ahsan H., Graziano J.H., Hussein I. and Ahmed K.M. (2003) Spatial variability of arsenic in 6000 tube wells in a 25 km<sup>2</sup> area of bangladesh, *Water Resources Research*, 39, 1140.
- Wolpert R.L., Steinberg L.J. and Reckhow K.H. (1993) Bayesian decision support using environmental transport-and-fate models (wth discussion), *Case Studies in Bayesian Statistics. Lecture Notes in Statist.*, 83, 241–293.